Exercícios de Cálculo II

UnB-Departamento de Matemática – Turma 08– 28/08/2025

17) Para quais valores de x as seguintes séries convergem? Encontre a *soma* da série para esses valores.

$$(a)\ \sum_{n=1}^{\infty}(-5)^n x^n \qquad (b)\ \sum_{n=1}^{\infty}(x+2)^n \qquad (c)\ \sum_{n=0}^{\infty}\frac{(x-2)^n}{3^n} \qquad (d)\ \sum_{n=0}^{\infty}\frac{\operatorname{sen}^n x}{3^n} \qquad (e)\ \sum_{n=0}^{\infty}\frac{2^n}{x^n}$$

18) Escreva a soma parcial s_n da série

$$\sum_{n=1}^{\infty}\ln(1+\frac{1}{n}).$$

Calcule a soma dos n termos de s_n. Essa série converge? O termo geral da série tende a zero? *Sugestão:* $\ln(a/b) = \ln a - \ln b$.

19) Se a soma parcial s_n de uma série $\sum_{n=1}^{\infty} a_n$ é dada por $s_n = (n-1)/(n+1)$, determine o termo geral a_n e a *soma* dessa série.

20) Verifique que

$$\sum_{n=0}^{\infty}(-1)^n a^n = 1 - a + a^2 - a^3 + \cdots = \frac{1}{1+a}, \quad |a| < 1.$$

Sugestão: substitua x por $-a$ em $1 + x + x^2 + x^3 + \cdots + x^n = (1 - x^{n+1})/(1 - x)$.

21) Determine o valor de c tal que $\sum_{n=2}^{\infty}(1+c)^{-n} = 2$.

22) Determine se as seguintes séries convergem ou divergem.

$$(a)\ \sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{n}} \qquad (b)\ \sum_{n=1}^{\infty}\frac{1}{n^5} \qquad (c)\ \sum_{n=1}^{\infty}\frac{1}{(2n+1)^3} \qquad (d)\ \sum_{n=1}^{\infty}\frac{1}{\sqrt{n+4}} \qquad (e)\ \sum_{n=1}^{\infty}\frac{n}{n^2+1}$$

$$(f)\ \sum_{n=1}^{\infty}ne^{-n^3} \qquad (g)\ \sum_{n=1}^{\infty}\frac{1}{n^{\sqrt{2}}} \qquad (h)\ \sum_{n=1}^{\infty}\frac{\sqrt{n}+4}{n^2} \qquad (i)\ \sum_{n=1}^{\infty}\frac{n^2}{n^3+1} \qquad (j)\ \sum_{n=1}^{\infty}\frac{1}{n^2+4}$$

$$(k)\ \sum_{n=3}^{\infty}\frac{3n-4}{n^2-2n} \qquad (l)\ \sum_{n=2}^{\infty}\frac{\ln n}{n^3} \qquad (m)\ \sum_{n=2}^{\infty}\frac{1}{n\ln n} \qquad (n)\ \sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2} \qquad (o)\ \sum_{n=1}^{\infty}\frac{e^{1/n}}{n^2}$$

23) Decida se as seguintes séries convergem:

$$(a)\ 1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots \qquad (b)\ 1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots \qquad (c)\ \sum_{n=1}^{\infty}\frac{\cos n\pi}{\sqrt{n}}$$

24) Decida se as seguintes séries convergem ou divergem:

$$(a)\ \sum_{n=1}^{\infty}\frac{n}{2n^3+1} \qquad (b)\ \sum_{n=2}^{\infty}\frac{n^3}{n^4-1} \qquad (c)\ \sum_{n=1}^{\infty}\frac{n+1}{n\sqrt{n}} \qquad (d)\ \sum_{n=1}^{\infty}\frac{6^n}{5^n-1} \qquad (e)\ \sum_{n=1}^{\infty}(1+\frac{1}{n})^2e^{-n}$$

$$(f)\ \sum_{k=1}^{\infty}\frac{k\operatorname{sen}^2 k}{1+k^3} \qquad (g)\ \sum_{k=1}^{\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}} \qquad (h)\ \sum_{n=1}^{\infty}\frac{\arctan n}{n^{1,2}} \qquad (i)\ \sum_{n=1}^{\infty}\frac{1+4^n}{1+3^n} \qquad (j)\ \sum_{k=1}^{\infty}\frac{\ln k}{k}$$

$$(k)\ \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n+1} \qquad (l)\ \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\ln(n+4)} \qquad (m)\ \sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{2n+1} \qquad (n)\ \sum_{n=1}^{\infty}(-1)^n\frac{n}{\sqrt{n^3+2}}$$

25) O que você pode dizer a respeito da série $\sum_{n=1}^{\infty} a_n$ que satisfaz:

$$(a)\ \lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = 8 \qquad (b)\ \lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = 0,8 \qquad (c)\ \lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = 1 \qquad (d)\ \lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = \frac{1}{8}$$

26) Decida se cada série a seguir é *absolutamente* convergente, *condicionalmente* convergente, ou divergente.

$$(a)\ \sum_{n=1}^{\infty}\frac{(-2)^n}{n^2} \quad (b)\ \sum_{n=1}^{\infty}\frac{n}{5^n} \quad (c)\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+4} \quad (d)\ \sum_{n=0}^{\infty}\frac{(-1)^n}{5n+1} \quad (e)\ \sum_{n=0}^{\infty}\frac{(-3)^n}{(2n+1)!}$$

$$(f)\ \sum_{k=1}^{\infty}\frac{k2^k}{3^k} \quad (g)\ \sum_{n=1}^{\infty}\frac{n!}{100^n} \quad (h)\ \sum_{n=1}^{\infty}(-1)^n\frac{(1,1)^n}{n^4} \quad (i)\ \sum_{n=1}^{\infty}(-1)^n\frac{n}{\sqrt{n^2+2}} \quad (j)\ \sum_{n=1}^{\infty}\frac{\operatorname{sen} 4n}{4^n}$$

$$(k)\ \sum_{n=1}^{\infty}(-1)^n\frac{e^{1/n}}{n^3} \quad (l)\ \sum_{n=1}^{\infty}\frac{10^n}{(n+1)4^{2n+1}} \quad (m)\ \sum_{n=1}^{\infty}\frac{n^{10}}{(-10)^{n+1}} \quad (n)\ \sum_{n=1}^{\infty}\frac{(-1)^n\arctan n}{n^2}$$

$$(o)\ \sum_{n=1}^{\infty}\frac{3-\cos n}{n^{2/3}-2} \quad (p)\ \sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n} \quad (q)\ \sum_{n=1}^{\infty}\frac{n!}{n^n} \quad (r)\ \sum_{n=1}^{\infty}\frac{\cos(n\pi/3)}{n!} \quad (s)\ \sum_{n=1}^{\infty}(\frac{n^2+1}{2n^2+1})^n$$

$$(t)\ \sum_{n=1}^{\infty}\frac{(-2)^n}{n^n} \quad (u)\ \sum_{n=1}^{\infty}(1+\frac{1}{n})^{n^2} \quad (v)\ \sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^n} \quad (x)\ \sum_{n=1}^{\infty}\frac{n^{10}10^n}{n!} \quad (y)\ \sum_{n=1}^{\infty}\frac{2^{n^2}}{n!} \quad (z)\ \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$$

27) Decida se a série a seguir é *absolutamente* convergente, *condicionalmente* convergente, ou divergente.

$$1-\frac{1\cdot 3}{3!}+\frac{1\cdot 3\cdot 5}{5!}-\frac{1\cdot 3\cdot 5\cdot 7}{7!}+\cdots+(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{(2n-1)!}+\cdots$$

28) Os termos da série $\sum_{n=1}^{\infty}a_n$ são definidos recursivamente. Determine se a série converge ou diverge:

$$(a)\ a_1=2,\quad a_{n+1}=\frac{5n+1}{4n+3}a_n \qquad (b)\ a_1=1,\quad a_{n+1}=\frac{2+\cos n}{\sqrt{n}}a_n$$

29) Obtenha o raio de convergência e o intervalo de convergência das séries:

$$(a)\ \sum_{n=1}^{\infty}(-1)^n n x^n \quad (b)\ \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{\sqrt[3]{n}} \quad (c)\ \sum_{n=1}^{\infty}\frac{x^n}{2n-1} \quad (d)\ \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n^2} \quad (e)\ \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$(f)\ \sum_{n=0}^{\infty}\frac{(x-2)^n}{n^2+1} \quad (g)\ \sum_{n=1}^{\infty}\frac{10^n x^n}{n^3} \quad (h)\ \sum_{n=1}^{\infty}\frac{3^n(x+4)^n}{\sqrt{n}} \quad (i)\ \sum_{n=2}^{\infty}(-1)^n\frac{x^n}{4^n\ln n}$$

$$(j)\ \sum_{n=1}^{\infty}(-1)^n\frac{n^2x^n}{2^n} \quad (k)\ \sum_{n=1}^{\infty}\frac{(-3)^n}{n\sqrt{n}}x^n \quad (l)\ \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \quad (m)\ \sum_{n=1}^{\infty}\frac{(2x-1)^n}{5^n\sqrt{n}}$$

Respostas: C=converge; D=diverge; CA=converge absolutamente; CC=converge condicionalmente.

17) **(a)** $|x|<1/5$; **(b)** $-3<x<-1$; **(c)** $-1<x<5$; **(d)** todo x real; **(e)** $|x|>2$;
18) $s_n=\ln(n+1)$;
19) $a_n=2/n(n+1)$; $s=1$;
21) $(-1+\sqrt{3})/2$; $(-1-\sqrt{3})/2$ não serve (por quê?);
22) **(a)** D; **(b)** C; **(c)** C; **(d)** D; **(e)** D; **(f)** C; **(g)** C; **(h)** C; **(i)** D; **(j)** C; **(k)** D; **(l)** C; **(m)** D; **(n)** C; **(o)** C;
23) **(a)** D; **(b)** CA; **(c)** C;
24) **(a)** C; **(b)** D; **(c)** D; **(d)** D; **(e)** C; **(f)** C; **(g)** CA; **(h)** C; **(i)** D; **(j)** D; **(k)** C; **(l)** C; **(m)** D; **(n)** C;
25) **(a)** D; **(b)** CA; **(c)** inconclusivo; **(d)** CA;
26) **(a)** D; **(b)** CA; **(c)** CC; **(d)** CC; **(e)** CA; **(f)** CA; **(g)** D; **(h)** D; **(i)** D; **(j)** CA; **(k)** CA; **(l)** CA; **(m)** CA; **(n)** CA; **(o)** D; **(p)** CC; **(q)** CA; **(r)** CA; **(s)** CA; **(t)** CA; **(u)** D; **(v)** CA; **(x)** CA; **(y)** D; **(z)** CC;
27) CA;
28) **(a)** D; **(b)** CA;
29) **(a)** $R=1$; $|x|<1$ **(b)** $R=1$; $-1<x\leq 1$; **(c)** $R=1$; $-1\leq x<1$ **(d)** $R=1$; $|x|\leq 1$ **(e)** $R=\infty$; todo x real; **(f)** $R=1$; $1\leq x\leq 3$ **(g)** $R=1/10$; $|x|\leq 1/10$ **(h)** $R=1/3$; $|x+4|<1/3$ **(i)** $R=4$; $-4<x\leq 4$ **(j)** $R=2$; $|x|<2$ **(k)** $R=1/3$; $-1/3\leq x\leq 1/3$ **(l)** $R=\infty$; $x\in\mathbb{R}$ **(m)** $R=5/2$; $-2\leq x<3$